Revelations: A Decidable Class of POMDPs with Omega-Regular Objectives

Marius Belly[1], Nathanaël Fijalkow[1], Hugo Gimbert[1], Florian Horn[2], Guillermo A. Pérez[3], Pierre Vandenhove[1]

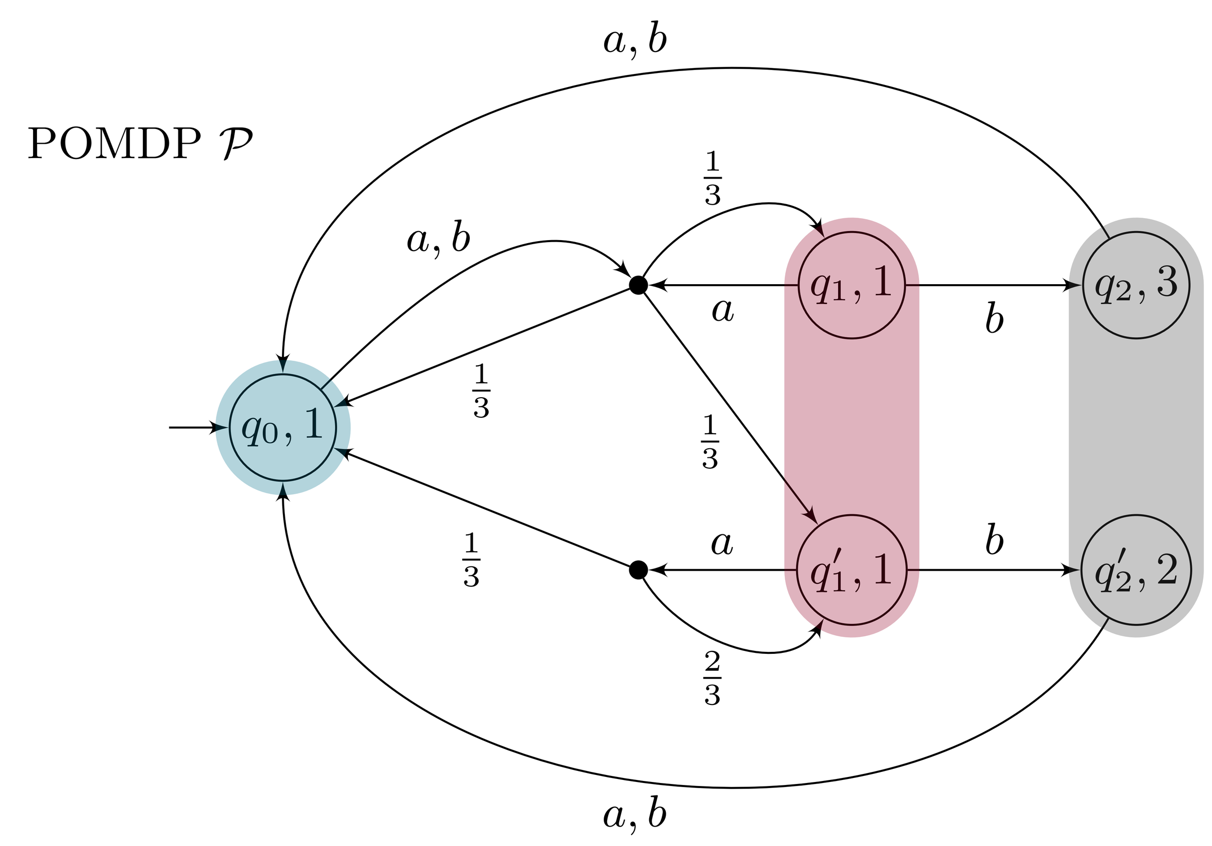
[1]CNRS, LaBRI, Université de Bordeaux, France [2]CNRS, IRIF, Université de Paris, France
[3]University of Antwerp – Flanders Make, Antwerp, Belgium

Outline

- **Partially observable Markov decision processes** (POMDPs):
  - stochasticity,
  - nondeterminism,
  - **uncertainty** about the actual state.
- Strategy synthesis is **undecidable** in general… Approach in this paper:
  - **Two decidable subclasses** with restrictions about **information loss**.
  - **Natural algorithm** that applies to this class.

Partially Observable Markov Decision Processes (POMDPs)

A **POMDP** is described by **states** Q, **initial state** q_0, **actions** Act, **observations** Obs. Running example:



Strategies are functions $(\mathrm{Act} \times \mathrm{Obs})^* \to \mathcal{D}(\mathrm{Act})$.

Classical Objectives: Very Undecidable!

- Function $p\colon Q \to \{0, \ldots, d\}$ assigning **priorities** to **states**.
- **Parity objective**: the **maximal** priority seen infinitely often is **even**.
- Common subclasses:
  - **Büchi**: $p\colon Q \to \{1, 2\}$: something good (2) occurs infinitely often,
  - **coBüchi**: $p\colon Q \to \{0, 1\}$: something bad (1) occurs finitely often.

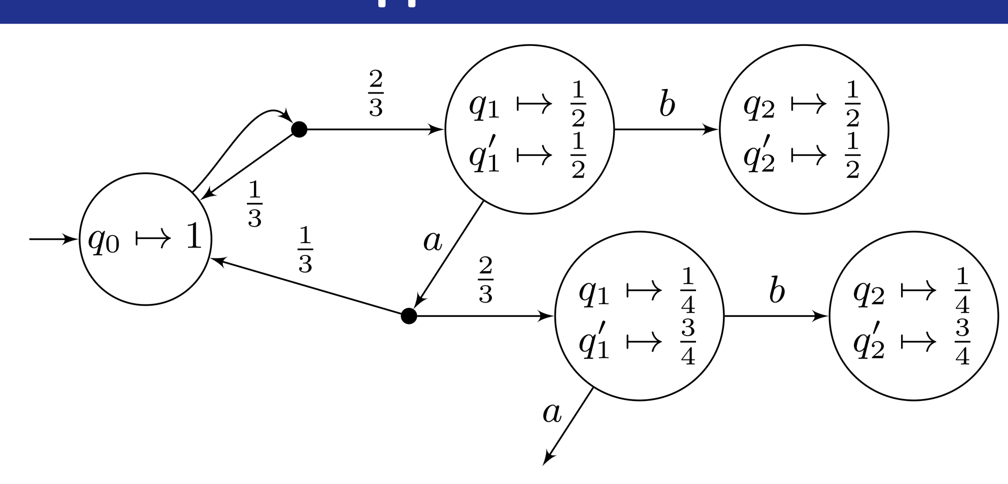
There is a (non-trivial) **almost-sure winning strategy** in $\mathcal{P}$. Can you find it?

Existing decidability results for almost-sure strategies [1, 2]

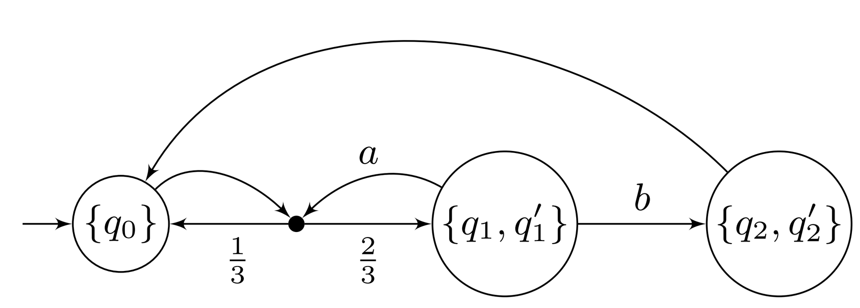
- Almost-sure **reachability**, **safety**, and **Büchi** are **EXPTIME-complete**.
- Almost-sure **coBüchi** (and therefore **parity**) are **undecidable**.

Beliefs vs. Belief Supports

POMDPs induce **infinite belief MDPs**:



Make it **finite**?
Only keep belief **supports**:



When does the analysis of the belief **support** MDP suffice?
In general, neither sound nor complete…

References

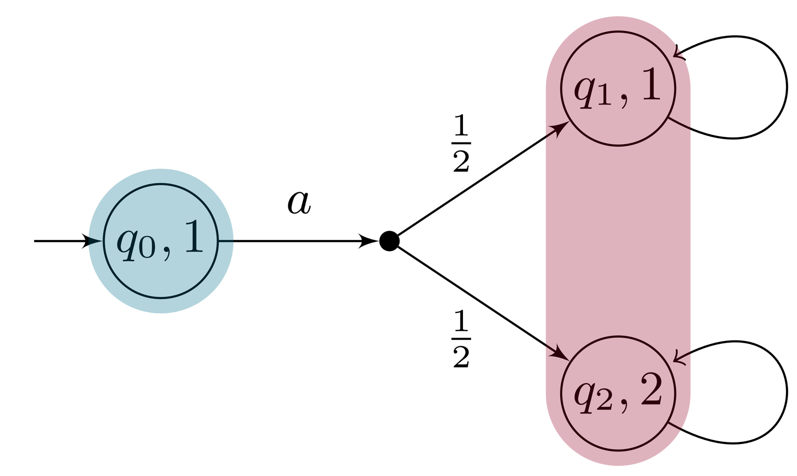
[1] Christel Baier, Marcus Größer, and Nathalie Bertrand. Probabilistic ω-automata. *J. ACM*, 59(1):1:1–1:52, 2012.
[2] Krishnendu Chatterjee, Martin Chmelik, and Mathieu Tracol. What is decidable about partially observable Markov decision processes with ω-regular objectives. *Journal of Computer and System Sciences*, 82(5):878–911, 2016.

Revealing Property #1: Weak Revelations

1. Weak revelations

A POMDP is **weakly revealing** if for all strategies, almost surely, the **current state can be known** infinitely often.

- POMDP $\mathcal{P}$ is weakly revealing: q_0 is almost surely visited infinitely often, no matter the strategy.
- This POMDP is not:



- When a **revealing history** happens, the finite belief **support** MDP contains as much information as the infinite belief MDP: $\{q_0\} \approx q_0 \mapsto 1$

Results for Weakly Revealing POMDPs

Soundness and completeness for priorities $\{0, 1, 2\}$

Almost-sure winning strategy in **POMDP** $\mathcal{P}$
$\iff$
Almost-sure winning strategy in the **belief support MDP** of $\mathcal{P}$.

Analysing the belief support MDP is **sound** and **complete** for parity $\{0, 1, 2\}$.

Decidability of weakly revealing POMDPs

Almost-sure **parity** $\{0, 1, 2\}$ for **weakly revealing** POMDPs is EXPTIME-complete.

Algorithm: solve the **belief support MDP** $\rightsquigarrow$ in EXPTIME.

However, almost-sure **parity** $\{1, 2, 3\}$ is still **undecidable**…
The belief support MDP is of no help in $\mathcal{P}$.

Revealing Property #2: Strong Revelations

2. Strong revelations

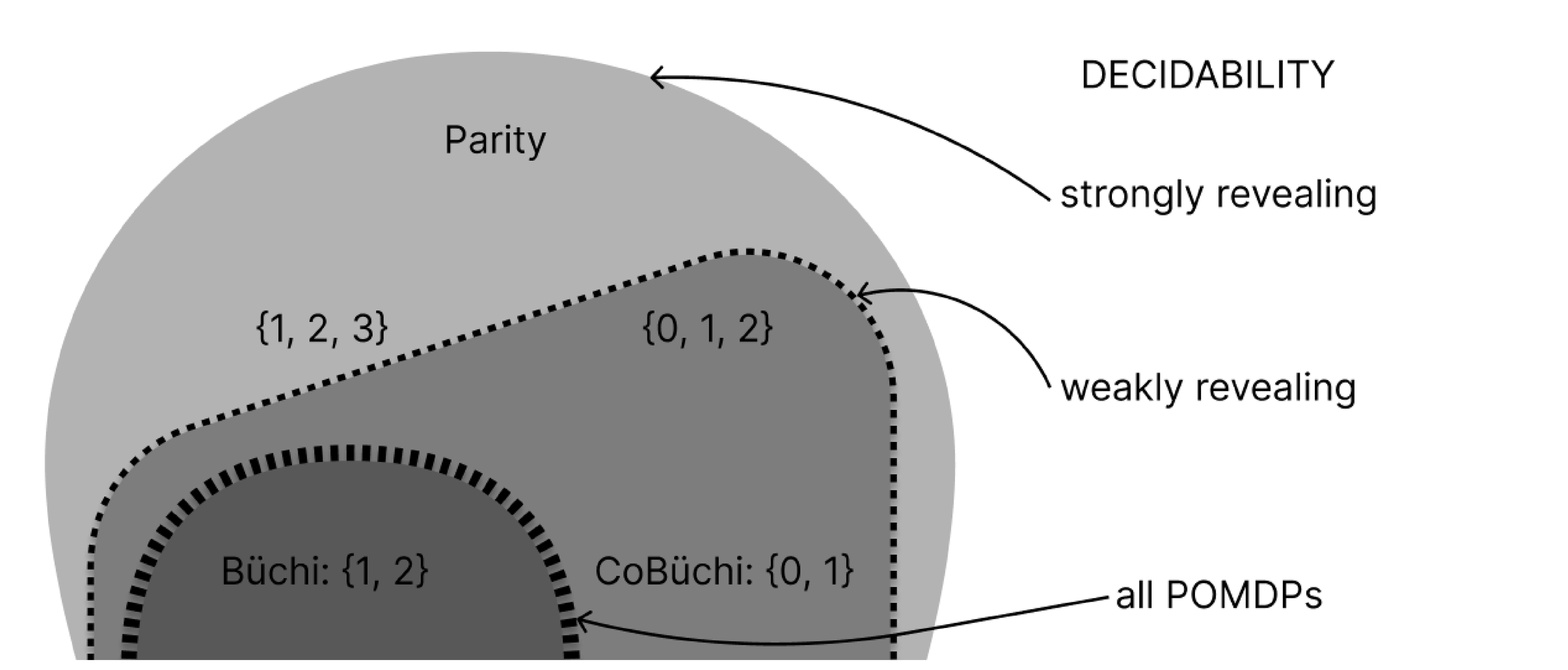
A POMDP is **strongly revealing** if for every transition $q \xrightarrow{a} q'$, there is a non-zero probability of "**observing** q'".

POMDP $\mathcal{P}$ is **not** strongly revealing: $q_1 \xrightarrow{a} q_1'$ is a possible transition, but nothing can reveal q_1' with certainty.

Decidability of strongly revealing POMDPs

Almost-sure **parity** for **strongly revealing** POMDPs is EXPTIME-complete.

Summary: Decidability of Revealing POMDPs



- Decidability frontier when we move to **games**: **games with partial observation** are still **undecidable** for coBüchi under **strong revelations**!
- **Implementation of the algorithms** at `https://github.com/gaperez64/pomdps-reveal`.