Arena-Independent Finite-Memory Strategies

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Outline

Strategy synthesis for two-player turn-based games

Design **optimal** controllers for systems interacting with an **antagonistic** environment.

"Optimal" w.r.t. an objective or a specification.

Goal: interest in "simple" controllers

Finite-memory determinacy: when do **finite-memory** controllers suffice?

Inspiration

Results by Gimbert and Zielonka[1,2] about **memoryless** determinacy.

[1] Gimbert and Zielonka, "Games Where You Can Play Optimally Without Any Memory", 2005.

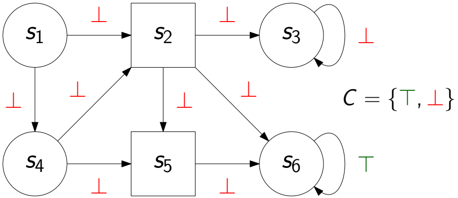
[2] Gimbert and Zielonka, "Pure and Stationary Optimal Strategies in Perfect-Information Stochastic Games with Global Preferences", 2009.

1 Memoryless determinacy

2 The need for memory

3 Arena-independent finite memory

Two-player turn-based zero-sum games on graphs



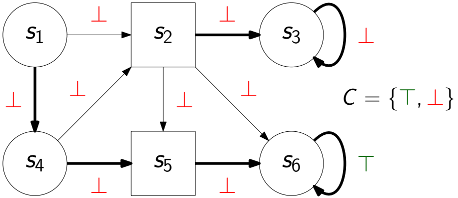
- **Finite** two-player **arenas**: S_1 ($\bigcirc$, for $\mathcal{P}_1$) and S_2 ($\Box$, for $\mathcal{P}_2$), edges E.
- Set C of **colors**. Edges are colored.
- "Objectives" given by **preference relations** $\sqsubseteq \subseteq C^\omega \times C^\omega$ (total preorder). **Zero-sum**.
- A **strategy** for $\mathcal{P}_i$ is a (partial) function $\sigma \colon E^* \to E$.

Memoryless determinacy

Question

Given a preference relation, do “simple” strategies suffice to play optimally in all arenas?

A strategy σ of $\mathcal{P}_i$ is *memoryless* if it is a function ~~E^*~~ $S_i \to E$.



E.g., for reachability, **memoryless** strategies suffice to play optimally.
Also suffice for safety, Büchi, co-Büchi, parity, mean payoff, energy, average-energy. . .

Memoryless determinacy

Good understanding of memoryless determinacy:

- **sufficient** conditions to guarantee memoryless optimal strategies for **both** players.[3,4]
- **sufficient** conditions to guarantee memoryless optimal strategies for **one** player.[5,6,7]
- **characterization of the preference relations admitting optimal memoryless strategies for both players.**[8]

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[3] Gimbert and Zielonka, "When Can You Play Positionally?", 2004.

[4] Aminof and Rubin, "First-cycle games", 2017.

[5] Kopczyński, "Half-Positional Determinacy of Infinite Games", 2006.

[6] Gimbert, "Pure Stationary Optimal Strategies in Markov Decision Processes", 2007.

[7] Gimbert and Kelmendi, "Two-Player Perfect-Information Shift-Invariant Submixing Stochastic Games Are Half-Positional", 2014.

[8] Gimbert and Zielonka, "Games Where You Can Play Optimally Without Any Memory", 2005.

Gimbert and Zielonka's characterization

Let $\sqsubseteq$ be a preference relation. One of the two main results:

One-to-two-player memoryless lift[9]

If

- in all **one-player** arenas of $\mathcal{P}_1$, $\mathcal{P}_1$ has an optimal memoryless strategy,
- in all **one-player** arenas of $\mathcal{P}_2$, $\mathcal{P}_2$ has an optimal memoryless strategy,

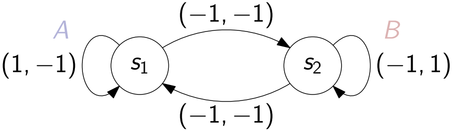
then both players have an optimal memoryless strategy in all **two-player** arenas.

Extremely useful in practice. Very easy to recover memoryless determinacy of, e.g., parity and mean-payoff games.

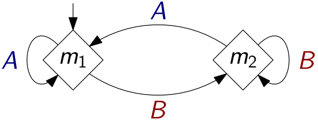
[9] Gimbert and Zielonka, "Games Where You Can Play Optimally Without Any Memory", 2005.

The need for memory

Memoryless strategies do not always suffice.



- Büchi(A) $\wedge$ Büchi(B): requires **finite memory**.



- Mean payoff ≥ 0 in both dimensions: requires **infinite memory**.[10]

$\rightsquigarrow$ **Combinations of objectives** often require memory.

[10] Chatterjee, Doyen, et al., "Generalized Mean-payoff and Energy Games", 2010.

An attempt at lifting [GZ05] to FM determinacy

- Lack of a good understanding of finite-memory determinacy.
- **Related work**: sufficient properties to preserve FM determinacy in Boolean combinations of objectives.[11]
- Our approach:

Hope: extend Gimbert and Zielonka's results

One-to-two-player lift for ~~memoryless~~ **finite-memory** determinacy?

[11] Le Roux, Pauly, and Randour, "Extending Finite-Memory Determinacy by Boolean Combination of Winning Conditions", 2018.

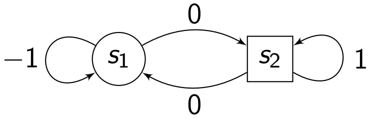
Counterexample to our hope

Let $C \subseteq \mathbb{Z}$. $\mathcal{P}_1$ wants to achieve a play $\pi = c_1 c_2 \ldots \in C^\omega$ s.t.

$$\limsup_n \sum_{i=1}^{n} c_i = +\infty \quad \text{or} \quad \exists^\infty n, \sum_{i=1}^{n} c_i = 0.$$

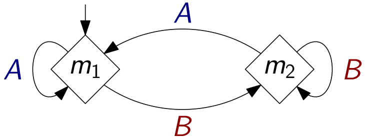
Optimal **FM** strategies in **one-player** arenas...

... not in **two-player** arenas: here, $\mathcal{P}_1$ wins but needs **infinite memory**.

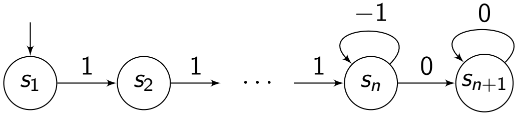


Distinction between the examples

- For Büchi(A) $\wedge$ Büchi(B), this structure suffices **for all** arenas for $\mathcal{P}_1$.



- The counterexample fails because in one-player arenas, the size of the memory is **dependent on the size of the arena**.



In this arena, $\mathcal{P}_1$ needs n memory states to win.

Arena-independent finite memory

Observation: for many objectives, one **fixed memory structure** suffices **for all arenas**.

"For all $\mathcal{A}$, does there exist $\mathcal{M}$...?"
$\rightarrow$ **"Does there exist $\mathcal{M}$, for all $\mathcal{A}$...?"**

Method: reproducing the approach of Gimbert and Zielonka **given an "arena-independent" memory structure $\mathcal{M}$**.

Characterization of arena-independent determinacy

Let $\sqsubseteq$ be a preference relation and $\mathcal{M}_1$, $\mathcal{M}_2$ be memory structures.

One-to-two-player arena-independent lift[12]

If

- in all **one-player** arenas of $\mathcal{P}_1$, $\mathcal{P}_1$ has an optimal strategy with memory $\mathcal{M}_1$,
- in all **one-player** arenas of $\mathcal{P}_2$, $\mathcal{P}_2$ has an optimal strategy with memory $\mathcal{M}_2$,

then both players have an optimal strategy in all **two-player** arenas with memory $\mathcal{M}_1 \times \mathcal{M}_2$.

In short: the study of **one-player arenas** is sufficient to determine whether playing with **arena-independent finite memory** suffices.

[12] Bouyer, Le Roux, et al., "Games Where You Can Play Optimally with Arena-Independent Finite Memory", 2020.

Applicability and limits

- **Applies to** objectives with optimal **arena-independent** strategies:
  - generalized reachability,[13]
  - generalized parity,[14]
  - window parity,[15]
  - lower- and upper-bounded (multi-dimensional) energy games.[16,17]

- **Does not apply to**, e.g., multi-dimension lower-bounded energy objectives:[18] the size of the finite memory depends on the arena.

[13] Fijalkow and Horn, "The surprizing complexity of reachability games", 2010.

[14] Chatterjee, Henzinger, and Piterman, "Generalized Parity Games", 2007.

[15] Bruyère, Hautem, and Randour, "Window parity games: an alternative approach toward parity games with time bounds", 2016.

[16] Bouyer, Markey, et al., "Average-energy games", 2018.

[17] Bouyer, Hofman, et al., "Bounding Average-Energy Games", 2017.

[18] Chatterjee, Randour, and Raskin, "Strategy synthesis for multi-dimensional quantitative objectives", 2014.

Results about **stochastic** games

Let $\sqsubseteq$ be a preference relation and $\mathcal{M}_1$, $\mathcal{M}_2$ be memory structures.

One-to-two-player **stochastic** lift[19]

If

- in all **one-player** **stochastic** arenas (i.e., MDPs) of $\mathcal{P}_1$, $\mathcal{P}_1$ has a **pure** optimal strategy with memory $\mathcal{M}_1$,
- in all **one-player** **stochastic** arenas (i.e., MDPs) of $\mathcal{P}_2$, $\mathcal{P}_2$ has a **pure** optimal strategy with memory $\mathcal{M}_2$,

then both players have a **pure** optimal strategy in all **two-player** **stochastic** arenas with memory $\mathcal{M}_1 \times \mathcal{M}_2$.

[19] Bouyer, Oualhadj, et al., "Arena-Independent Finite-Memory Determinacy in Stochastic Games", 2021.

Summary

Key observation: **arena-independent** memory often suffices.

Contributions

- **One-to-two-player lift** in deterministic and stochastic games.
- Characterization of **arena-independent** finite-memory determinacy.

Ongoing work

- Understand the arena-**dependent** case.
- Similar one-to-two-player lift for **infinite** arenas.

Thanks! Questions?