Reachability in Stochastic Hybrid Systems
[Ongoing Work]

Patricia Bouyer[1] Thomas Brihaye[2] Mickael Randour[2,3]
Cédric Rivière[2] **Pierre Vandenhove**[1,2,3]

[1]LSV, CNRS & ENS Paris-Saclay, Université Paris-Saclay, France
[2]UMONS – Université de Mons, Mons, Belgium
[3]F.R.S.-FNRS

September 19, 2019 – Highlights 2019, Warsaw

Outline

Goal

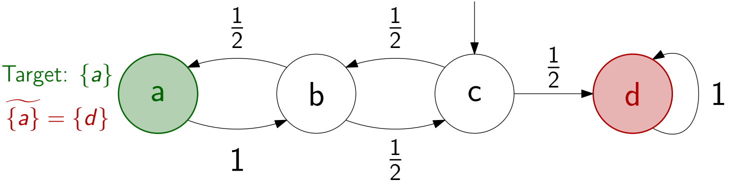
Identify a **decidable class** for **reachability** in **stochastic hybrid systems**.

Method

Follow an approach that has been successful for **infinite Markov chains**.

Reachability in infinite Markov chains

Let $\mathcal{M}$ be a countable Markov chain.



Let $B \subseteq S$ be a set of target states, $s \in S$ be an initial state.

Goal

Compute (or approximate) $\mathrm{Prob}_s^{\mathcal{M}}(\Diamond B)$.

We set

$$\widetilde{B} = \{s \in S \mid \mathrm{Prob}_s^{\mathcal{M}}(\Diamond B) = 0\}.$$

How to approximate the probability of reaching B?

Approximation procedure[1]

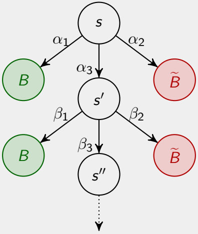
We define for $n \in \mathbb{N}$

$$p_n^{\text{Yes}} = \text{Prob}_s^{\mathcal{M}}(\Diamond_{\leq n} B), \qquad p_n^{\text{No}} = \text{Prob}_s^{\mathcal{M}}(\Diamond_{\leq n} \widetilde{B}).$$

Given a precision $\epsilon > 0$, we **compute**

$$p_1^{\text{Yes}} \leq \text{Prob}_s^{\mathcal{M}}(\Diamond B) \leq 1 - p_1^{\text{No}}$$
$$p_2^{\text{Yes}} \leq \text{Prob}_s^{\mathcal{M}}(\Diamond B) \leq 1 - p_2^{\text{No}}$$
$$\vdots$$

Until $(1 - p_n^{\text{No}}) - p_n^{\text{Yes}} < \epsilon$.

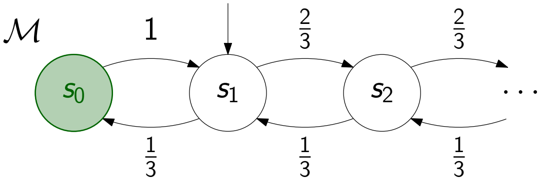


Always terminates?

[1] Iyer and Narasimha, "Probabilistic Lossy Channel Systems", 1997.

Counterexample: diverging random walk

The procedure does not terminate for this infinite Markov chain:



Initial state: s_1, target state: $B = \{s_0\} \implies \widetilde{B} = \emptyset$. For all n,

- $p_n^{\text{Yes}} = \text{Prob}_{s_1}^{\mathcal{M}}(\Diamond_{\leq n} B) \leq \text{Prob}_{s_1}^{\mathcal{M}}(\Diamond B) = \frac{1}{2}$.
- $p_n^{\text{No}} = \text{Prob}_{s_1}^{\mathcal{M}}(\Diamond_{\leq n} \widetilde{B}) = 0$.

$\rightsquigarrow$ For all n, $(1 - p_n^{\text{No}}) - p_n^{\text{Yes}} \geq \frac{1}{2} \dots$

Decisiveness

Let $\mathcal{M} = (S, P)$ be a countable Markov chain, $B \subseteq S$.

Decisiveness[2]

$\mathcal{M}$ is **decisive** w.r.t. $B \subseteq S$ if for all $s \in S$, $\mathrm{Prob}_s^{\mathcal{M}}(\Diamond B \vee \Diamond \widetilde{B}) = 1$.

Theorem[2]

If $\mathcal{M}$ is decisive w.r.t. B, then the approximation procedure is correct and **terminates**.

Decisiveness was extended to more general stochastic systems.[3]

Goal

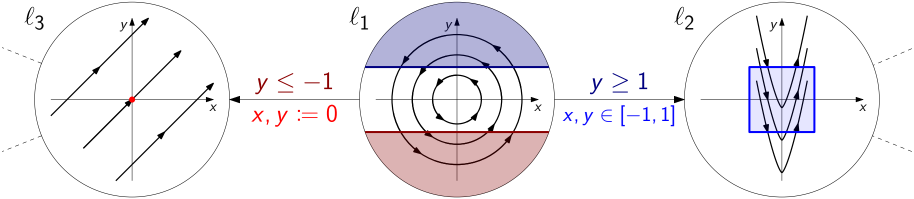
Identify classes of decisive **stochastic hybrid systems**.

---

[2]Abdulla, Ben Henda, and Mayr, "Decisive Markov Chains", 2007.

[3]Bertrand et al., "When are stochastic transition systems tameable?", 2018.

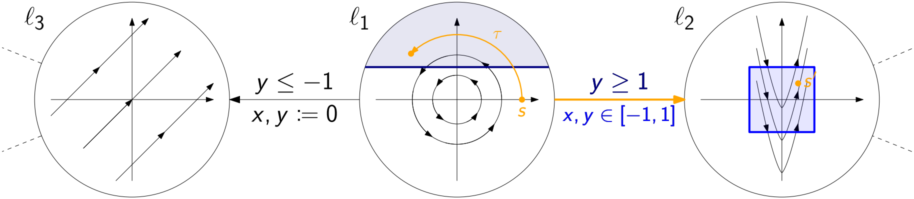
Hybrid systems (“discrete + continuous”)



- (L, E) is a **finite graph**.
- A number n of **continuous variables**
  $\rightsquigarrow$ states of the system are in $L \times \mathbb{R}^n$: **uncountable** state space!
- A **continuous dynamics** in each discrete location.
- A **guard** for each edge.
- A **reset** for each edge.

We replace the nondeterminism of hybrid systems with probability distributions on the:

- waiting time from a given state;
- edge choice;
- choice of a reset value.



$\rightsquigarrow$ **Stochastic** hybrid systems (**SHSs**)

Undecidability

Reachability (almost-sure and approximation) is **undecidable** for SHSs.

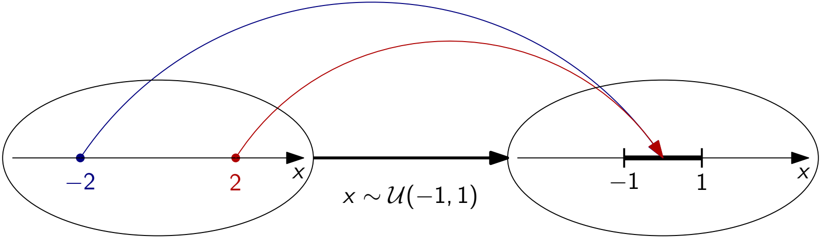
Goal

Find a subclass of SHSs for which reachability is decidable.

Making reachability decidable with strong resets

Strong reset[4] = reset that does not depend on the value of the variables.

Example: x follows a uniform dist. in $[x-1, x+1]$ **is not** a strong reset.
x follows a uniform distribution in $[-1, 1]$ **is** a strong reset.



Main result

SHSs with one strong reset per cycle are

- **decisive** w.r.t. any set of states;
- have a **finite abstraction**!

[4] Lafferriere, Pappas, and Sastry, "O-Minimal Hybrid Systems", 2000.

Conclusion: decidable classes of hybrid systems

Hybrid systems: existence of a finite bisimulation

- Timed automata;[5]
- Initialized rectangular hybrid systems;[6]
- O-minimal hybrid systems.[7]

Stochastic hybrid systems: decisiveness and finite abstraction

- Single-clock stochastic timed automata;[8]
- Reactive stochastic timed automata;[8]
- **Strongly-reset stochastic hybrid systems**.

$\rightsquigarrow$ Reachability is **decidable** under effectiveness assumptions.

[5] Alur and Dill, "Automata For Modeling Real-Time Systems", 1990.
[6] Henzinger et al., "What's Decidable about Hybrid Automata?", 1998.
[7] Lafferriere, Pappas, and Sastry, "O-Minimal Hybrid Systems", 2000.
[8] Bertrand et al., "When are stochastic transition systems tameable?", 2018.