Arena-Independent Finite-Memory Determinacy
[Ongoing work]

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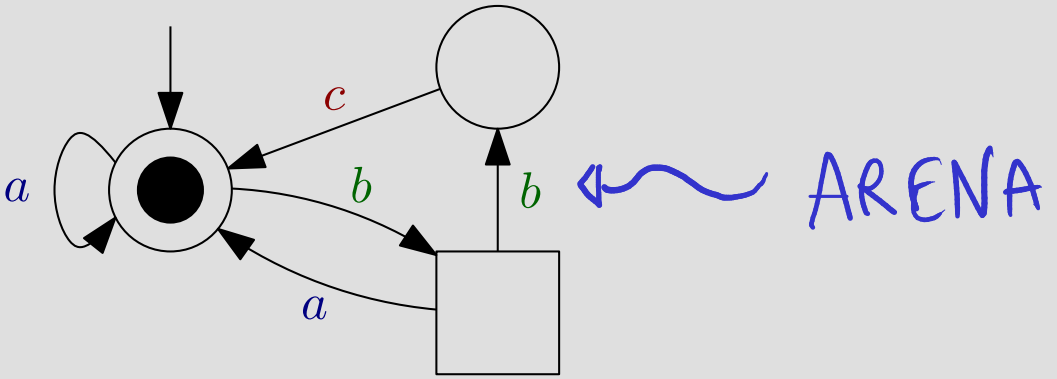
September 15, 2021 – Highlights 2021

Games on graphs

Zero-sum turn-based games on graphs



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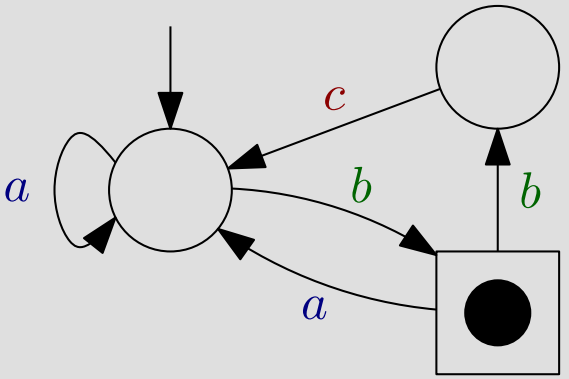
Zero-sum turn-based games on graphs

$C = \{a, b, c\}$



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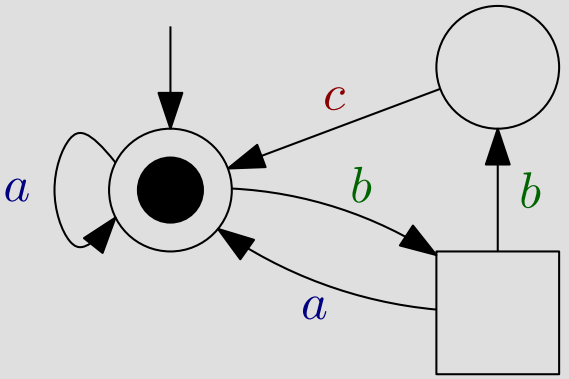

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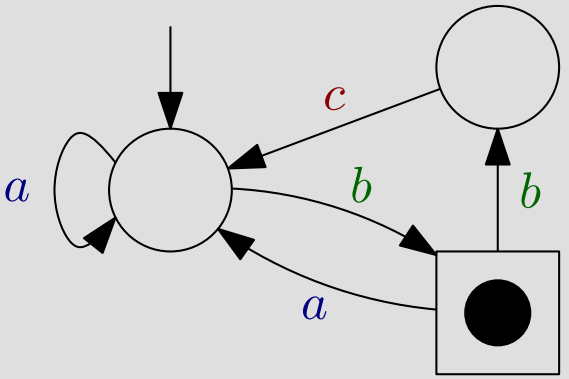

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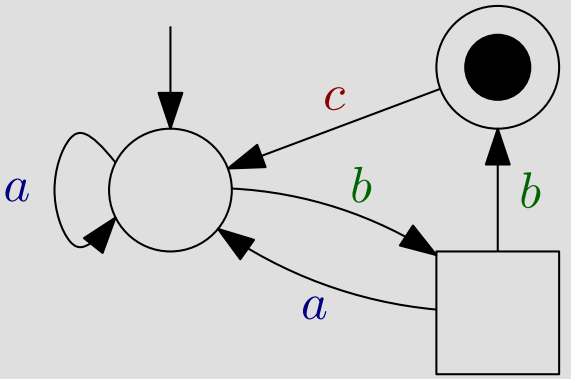

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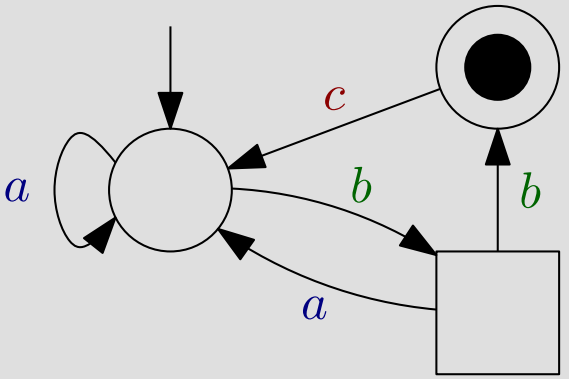

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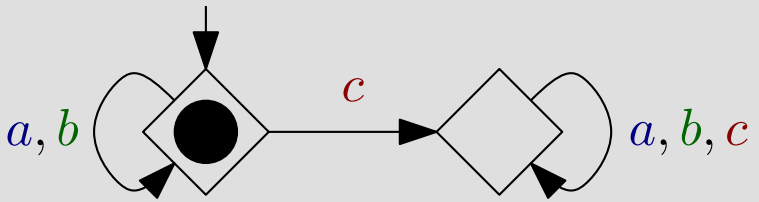
Strategy complexity

Given an objective, what information to remember about the past?



Memory structure $\approx$ automaton.

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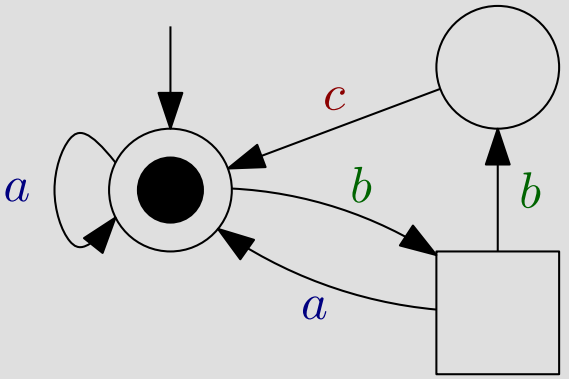

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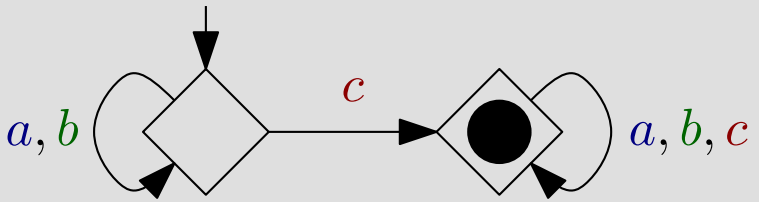

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Arena-independent finite-memory determinacy

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objectives for which a **unique finite memory structure** suffices to play optimally in **all arenas**.

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$\rightsquigarrow$ good trade-off between **applicability** (e.g., all ω-regular objectives, but not only) and **good properties**...

One-to-two-player lifts

When does two-player zero-sum memory determinacy reduce to **one-player** memory determinacy?

Arenas \ Strat. comp.	Memoryless	**Arena-ind. FM**	Mildly growing
Finite deterministic	[GZ05][1]	**[BLORV20]**[2]	[Koz21][3]
Finite stochastic	[GZ09][4]	**[BORV21]**[5]	
Infinite deterministic	[CN06,Kop08][6]	**New work**	

[1] Gimbert and Zielonka, "Games Where You Can Play Optimally Without Any Memory", 2005.

[2] Bouyer, Le Roux, et al., "Games Where You Can Play Optimally with Arena-Independent Finite Memory", 2020.

[3] Kozachinskiy, "One-to-Two-Player Lifting for Mildly Growing Memory", 2021.

[4] Gimbert and Zielonka, "Pure and Stationary Optimal Strategies in Perfect-Information Stochastic Games with Global Preferences", 2009.

[5] Bouyer, Oualhadj, et al., "Arena-Independent Finite-Memory Determinacy in Stochastic Games", 2021.

[6] Colcombet and Niwiński, "On the positional determinacy of edge-labeled games", 2006; Kopczyński, "Half-positional Determinacy of Infinite Games", 2008.

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Infinite arenas, memoryless strategies

Let $W \subseteq C^\omega$ be a **prefix-independent** objective.

Theorem[7]

If **memoryless strategies** suffice to play optimally for both players in **one-player infinite deterministic arenas**, then W is a **parity condition**.

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If **memoryless strategies** suffice to play optimally for both players in **one-player infinite deterministic arenas**, then W is a **parity condition**.

In other words: there exists $p\colon C \to \{0, \dots, n\}$ such that

$$w = c_1 c_2 \ldots \in W \iff \limsup_i p(c_i) \text{ is even.}$$

[7] Colcombet and Niwiński, "On the positional determinacy of edge-labeled games", 2006.

Infinite arenas, **arena-independent finite memory**

Let $W \subseteq C^\omega$ be a **prefix-independent** objective, $\mathcal{M}$ be a memory structure.

Theorem

If **memoryless strategies** (using memory $\mathcal{M}$) suffice to play optimally for both players in **one-player infinite deterministic arenas**, then W is (recognized by a parity automaton) **a parity condition**.

Corollary 1. New one-to-two-player lift.

Corollary 2. Strategic characterization of ω-regular objectives.

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Corollary 1. New one-to-two-player lift.

Corollary 2. Strategic characterization of ω-regular objectives.

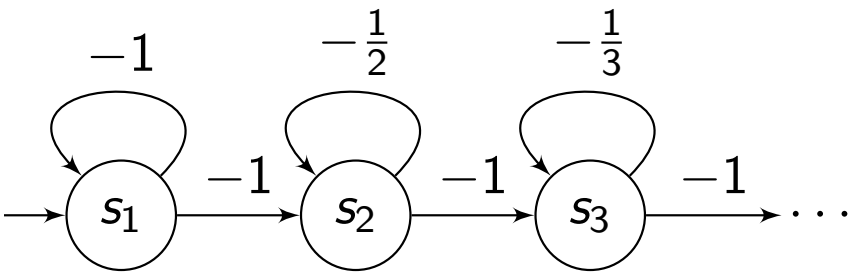
Thanks! Questions?

Appendix

Greater memory requirements in **infinite** arenas

Objective: get a mean payoff ≥ 0.

- Memoryless strategies suffice in finite (even stochastic) arenas.
- Infinite memory is required in one-player deterministic infinite arenas.[7]



$\rightsquigarrow$ Possible to get 0 at the limit **with infinite memory**.

[7] Puterman, *Markov Decision Processes: Discrete Stochastic Dynamic Programming*, 1994.