Games Where You Can Play Optimally with Arena-Independent Finite Memory

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Outline

Strategy synthesis for two-player turn-based games

Design optimal controllers for systems interacting with an antagonistic environment.

"Optimal" w.r.t. an objective or a specification.

Goal: interest in "simple" controllers

Finite-memory determinacy: when do finite-memory controllers suffice?

Inspiration

Results by Gimbert and Zielonka[1] about memoryless determinacy.

[1] Gimbert and Zielonka, "Games Where You Can Play Optimally Without Any Memory", 2005.

1. Memorylessness determinacy

2. The need for memory

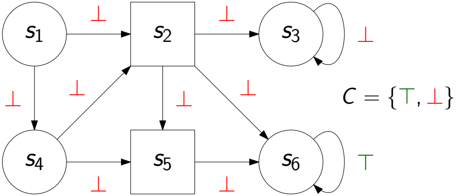
3. Arena-independent finite memory

1 Memoryless determinacy

2 The need for memory

3 Arena-independent finite memory

Two-player turn-based zero-sum games on graphs



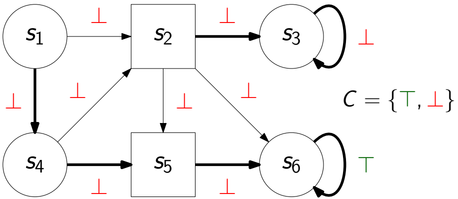
- Finite two-player arenas: S_1 (circles, for $\mathcal{P}_1$) and S_2 (squares, for $\mathcal{P}_2$), edges E.
- Set C of colors. Edges are colored.
- "Objectives" given by preference relations $\sqsubseteq \in C^\omega \times C^\omega$ (total preorder). Zero-sum, $\sqsubseteq^{-1}$.
- A strategy for $\mathcal{P}_i$ is a (partial) function $\sigma\colon E^* \to E$.

Memoryless determinacy

Question

Given a preference relation, do "simple" strategies suffice to play optimally in all arenas?

A strategy σ of $\mathcal{P}_i$ is *memoryless* if it is a function ~~E^*~~ $S_i \to E$.



E.g., for reachability, memoryless strategies suffice.
Also suffice for safety, Büchi, co-Büchi, parity, mean-payoff, energy, average-energy...

Memoryless determinacy

Good understanding of memoryless determinacy:

- **sufficient** conditions to guarantee memoryless optimal strategies for **both** players.[2,3]
- **sufficient** conditions to guarantee memoryless optimal strategies for **one** player.[4,5,6]
- **characterization of the preference relations admitting optimal memoryless strategies for both players.**[7]

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[2]Gimbert and Zielonka, "When Can You Play Positionally?", 2004.

[3]Aminof and Rubin, "First-cycle games", 2017.

[4]Kopczynski, "Half-Positional Determinacy of Infinite Games", 2006.

[5]Gimbert, "Pure Stationary Optimal Strategies in Markov Decision Processes", 2007.

[6]Gimbert and Kelmendi, "Two-Player Perfect-Information Shift-Invariant Submixing Stochastic Games Are Half-Positional", 2014.

[7]Gimbert and Zielonka, "Games Where You Can Play Optimally Without Any Memory", 2005.

Gimbert and Zielonka's characterization[8]

Let $\sqsubseteq$ be a preference relation. Two results:

1. Characterization of memoryless determinacy w.r.t. properties of $\sqsubseteq$.
2. Corollary:

One-to-two-player memoryless lifting

If

- in all **one-player** arenas of $\mathcal{P}_1$, $\mathcal{P}_1$ has an optimal memoryless strategy,
- in all **one-player** arenas of $\mathcal{P}_2$, $\mathcal{P}_2$ has an optimal memoryless strategy,

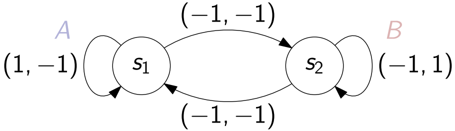
then both players have an optimal memoryless strategy in all **two-player** arenas.

Extremely useful in practice. Very easy to recover memoryless determinacy of, e.g., mean-payoff and parity games.

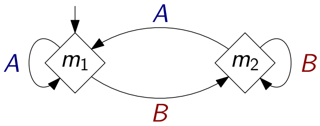
[8] Gimbert and Zielonka, "Games Where You Can Play Optimally Without Any Memory", 2005.

The need for memory

Memoryless strategies do not always suffice.



- Büchi(A) $\wedge$ Büchi(B): requires **finite memory**.



- Mean payoff ≥ 0 in both dimensions: requires **infinite memory**.[9]

$\rightsquigarrow$ **Combinations of objectives** usually require memory.

[9] Chatterjee, Doyen, et al., "Generalized Mean-payoff and Energy Games", 2010.

An attempt at lifting [GZ05] to FM determinacy

- Lack of a good understanding of finite-memory determinacy.
- **Related work**: sufficient properties to preserve FM determinacy in Boolean combinations of objectives.[10]
- Our approach:

Hope: extend Gimbert and Zielonka's results

One-to-two-player lifting for ~~memoryless~~ **finite-memory** determinacy.

[10] Le Roux, Pauly, and Randour, "Extending Finite-Memory Determinacy by Boolean Combination of Winning Conditions", 2018.

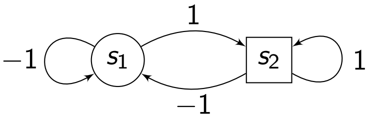
Counterexample

Let $C \subseteq \mathbb{Z}$. $\mathcal{P}_1$ wants to achieve a play $\pi = c_1 c_2 \ldots \in C^\omega$ s.t.

$$\limsup_n \sum_{i=0}^{n} c_i = +\infty \quad \text{or} \quad \exists^\infty n, \sum_{i=0}^{n} c_i = 0.$$

Optimal **FM** strategies in **one-player** arenas. . .

. . . but not in **two-player** arenas: $\mathcal{P}_1$ wins but needs **infinite memory**.



Intuition:

In one-player arenas, $\mathcal{P}_1$ can bound the memory he needs in advance.

In two-player arenas, $\mathcal{P}_2$ can generate arbitrarily long sequences.

1 Memoryless determinacy

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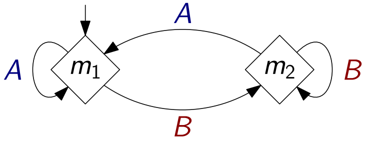
Arena-independent memory

- For Büchi(A) $\wedge$ Büchi(B), this structure suffices to play optimally on **all** arenas for $\mathcal{P}_1$.



- The counterexample fails because in one-player arenas, the size of the memory is **dependent on the size of the arena**.
- Observation: for many objectives, one **fixed memory structure** suffices **for all arenas**.

"For all $\mathcal{A}$, does there exist $\mathcal{M}$...?"
$\rightarrow$ **"Does there exist $\mathcal{M}$, for all $\mathcal{A}$...?"**

Method: reproducing the approach of Gimbert and Zielonka **given a memory structure** $\mathcal{M}$.

Characterization of arena-independent determinacy

Let $\sqsubseteq$ be preference relation, $\mathcal{M}$ be a memory structure.

1. Characterization of "playing with $\mathcal{M}$ is sufficient" in terms of properties of $\sqsubseteq$.
2. Corollary:

One-to-two-player lifting

If

- in all **one-player** arenas of $\mathcal{P}_1$, $\mathcal{P}_1$ has an optimal strategy with memory $\mathcal{M}_1$,
- in all **one-player** arenas of $\mathcal{P}_2$, $\mathcal{P}_2$ has an optimal strategy with memory $\mathcal{M}_2$,

then both players have an optimal strategy in all **two-player** arenas with memory $\mathcal{M}_1 \otimes \mathcal{M}_2$.

In short: the study of **one-player arenas** is sufficient to determine whether playing with arena-independent finite memory suffices.

Applicability and limits

- **Applies to** objectives with optimal **arena-independent** strategies:
  - generalized reachability,[11]
  - generalized parity,[12]
  - window parity,[13]
  - lower- and upper-bounded (multi-dimensional) energy games.[14,15]

- **Does not apply to**, e.g., multi-dimension lower-bounded energy objectives:[16] the size of the finite memory depends on the arena.

[11] Fijalkow and Horn, "The surprizing complexity of reachability games", 2010.

[12] Chatterjee, Henzinger, and Piterman, "Generalized Parity Games", 2007.

[13] Bruyère, Hautem, and Randour, "Window parity games: an alternative approach toward parity games with time bounds", 2016.

[14] Bouyer, Markey, et al., "Average-energy games", 2018.

[15] Bouyer, Hofman, et al., "Bounding Average-Energy Games", 2017.

[16] Chatterjee, Randour, and Raskin, "Strategy synthesis for multi-dimensional quantitative objectives", 2014.

Conclusion

Key observation: many objectives require arena-independent memory.

Contributions

- Characterization of arena-independent finite-memory determinacy.
- One-to-two-player lifting.
- Generalization of Gimbert and Zielonka's work.

Future work

Understand (arena-**dependent**) finite-memory determinacy through the study of one-player arenas.

Thanks!